

What expertise do teachers need to exploit the epistemic potential of multiple languages for mathematics learning?

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Abstract. Based on many decades of research on using multiple languages in mathematics classrooms, instructional approaches have been developed and promoted to exploit students' multilingual resources for mathematics teaching and learning. Some of these approaches go beyond the communicative potential (to increase all students' participation) and focus on the epistemic potential of deepening students' understanding of mathematical concepts. However, an implementation gap exists in that only a few mathematics teachers have started to exploit this epistemic potential in their classrooms. To overcome this implementation gap, specific professional development opportunities for multilanguage-responsive mathematics teaching are required. In this paper, we lay a foundation for developing such professional development opportunities by specifying what teachers may need to learn in order to respond appropriately to typical situational demands of eliciting multiple languages for their epistemic potential. For this, we extend the existing model of expertise for language-responsive mathematics teaching by refining already identified situational demands, with respect to the epistemic potential of multiple languages.

Keywords. Multilingual language resources, epistemic potential, teacher expertise, multilanguage-responsive mathematics teaching

1 Introduction

In most classrooms in the world, students speak more than one language, resulting in a long research tradition on mathematics learning in multilingual classrooms (Austin & Howson, 1979; Barwell et al., 2016). Research findings include insights into the political role of language choices and restrictions in classrooms (e.g., Austin & Howson, 1979; Trinick, 2014) or the higher valuing of one language over others (e.g., Meaney et al., 2017; Setati, 2005). Empirical classroom studies on the *communicative role* of multiple languages revealed that multilingual students use their languages in classrooms even when this is not officially allowed. However, when the use of multiple languages is allowed or encouraged, this can substantially contribute to more agency and higher participation of all students (Barwell et al., 2016; Norén, 2015). Furthermore, research revealed that the use of multiple languages can also take an *epistemic role* for enhancing students' mathematical thinking and learning. For example, using multiple languages for discussing mathematics can lead to improved mathematical reasoning (Dawe, 1985) and support the development of mathematical understanding (Barwell, 2020; Moschkovich, 2002; Planas, 2018; Prediger & Uribe, 2021). The focus of our project *ML² – Exploiting the Power of Multiple Languages for Mathematics Learning*, is the epistemic potential of multiple languages for conceptual understanding. Our aim is to design teaching approaches and PD opportunities for teachers to harness students' multiple language as resource to promote the conceptual understanding of mathematics. Although the project is situated in

Europe, there are implications for other multilingual mathematics classrooms, in Pacific countries, such as Australia, New Zealand and Papua New Guinea (see for example Clarkson & Thomas, 1993).

The findings on the communicative and epistemic potential of including multiple languages in mathematics teaching and learning have fuelled an increasingly widespread call for instructional approaches that build on students' multiple languages as resources for mathematics learning (Barwell 2020; Planas 2018; Ruíz 1984). Although these instructional approaches are widely acknowledged in educational policy documents (e.g., for Europe: Beacco et al. 2010), their implementation in mathematics classrooms remains limited, particularly with respect to the epistemic potential of multiple languages.

One reason for this implementation gap may be that there are limited professional development (PD) opportunities for mathematics teachers to learn about mathematics teaching that is responsive to students' multiple languages (Adler 2001; Clarkson & Thomas, 1993; Essien et al., 2016). Currently, language-related PD programs for European mathematics teachers have either a focus on subject-independent, second-language learning, including the promotion of multiple languages as resources with communicative potential (e.g., Lucas & Villegas, 2013), or on working on languages for mathematics learning in Language of Instruction (LoI), without much focus on the use of multiple languages (Hajer & Noren, 2017; Prediger, 2019),

Teachers are acknowledged as the most important contributors to providing mathematical learning opportunities, also in multilingual classrooms (Hansson, 2012). Yet, previous research in multilingual classrooms has revealed that teachers can be unprepared for making use of students' proficiency in different languages for learning mathematics and can be actively resistant to developing strategies that make use of students' language proficiency (Thomas, 1986; Aguirre et al., 2012). Some reasons for not utilizing students' proficiency in multiple languages can be related to orientations that frame languages other than the LoI as distracting students from learning mathematics (Xenofontos, 2016). The challenges for teachers to include multiple languages seem to be greater in super-diverse multilingual classrooms, where there are many non-shared languages (not shared between the students, and between the students and the teacher) than in classrooms with shared bilingualism (e.g., the Californian context in which many teachers and students share English and Spanish). However, it is exactly this super-diversity of many non-shared languages which is typical of many classrooms, not only in the European context (Sierens & van Avermaet, 2014; Vertovec, 2007).

The aim of this chapter is to discuss potential foundations for PD programs that enhance teachers' expertise in multilanguage-responsive teaching (a construct to be defined in the theory section) with the following question: *What practices and underlying orientations will best enable teachers to learn to exploit the epistemic potential of using multiple languages for mathematics learning?*

To investigate this question, we extend existing models about teaching language and mathematics integratively (focusing mainly in one language, the LoI) to include understandings of teaching through multiple languages (Section 2). By synthesizing the existing research on using multiple languages for developing mathematical understanding, we extend an existing framework of language-responsive teacher expertise (Prediger, 2019) and produce a list of productive orientations and assumptions that should be addressed and developed in PD programs to guide teachers' practices (Section 3).

2 Specified expertise for language-responsive mathematics teaching

Adler's (2001) early work on multilingual mathematics classrooms revealed how teachers who start to work on languages in mathematics education assume that two educational ambitions are in opposition to each other, that of promoting students' learning of the LoI and of building on multilingual language resources as a learning medium. In contrast, expert teachers on languages in mathematics education were able to integrate both educational ambitions, taking advantage of the potential of multiple languages to simultaneously support the mathematics learning and the learning of the LoI. Therefore, our approach emphasises the importance of *integrating both ambitions* by adopting the assumption that the epistemic potential of using multiple languages is best unfolded within language-responsive teaching practices (Prediger & Uribe, 2021; Schüler-Meyer et al., 2023).

We define *language-responsive mathematics teaching* as the teaching practices that deepen mathematics learning (in particular the development of conceptual understanding), by explicitly working with and enhancing students' academic language proficiency in the LoI (Lucas & Villegas, 2013; Prediger, 2019). In this section, we briefly present an existing conceptual framework on teacher expertise for language-responsive mathematics teaching. In Section 3, this framework is extended to include the use of multiple languages, to what we call teacher expertise in multilanguage-responsive teaching.

For a PD program targeting a particular expertise, it is important to unpack its long-term goals, particular with respect to specifying the first steps towards these goals. In our case, our PD program aims to develop teachers expertise towards expertise for (multi-)language-responsive mathematics teaching. In practice-based approaches (Ball & Forzani, 2009; Bromme 1992), the specification process starts by choosing recurring, situational demands of classroom teaching (briefly called *jobs*) and then proceeds with (empirically) identifying the typical *practices* that expert teachers enact to respond to these demands. This specification enables the design of learning opportunities for teachers to develop such expert practices (Prediger, 2019) by:

- (a) decomposing the practices into visible (patterned) actions, utterances and *pedagogical tools* (i.e., artefacts used to enact the practices, e.g., tasks, facilitation moves, activity structures, and manipulatives), and
- (b) unpacking the practices with respect to the underlying *orientations* (i.e., as content-related and more general beliefs that implicitly or explicitly guide the teachers' perceptions and job priorities) and knowledge *categories* used for perceiving and thinking.

Figure 1 shows the result of such a specification process, in a content map of expertise for language-responsive mathematics teaching. Starting from six jobs (top part), it lists the pedagogical tools, categories and orientations needed to respond to these six jobs with productive practices. To reduce complexity, this paper will neglect the categories and focus on jobs, practices, and orientations.

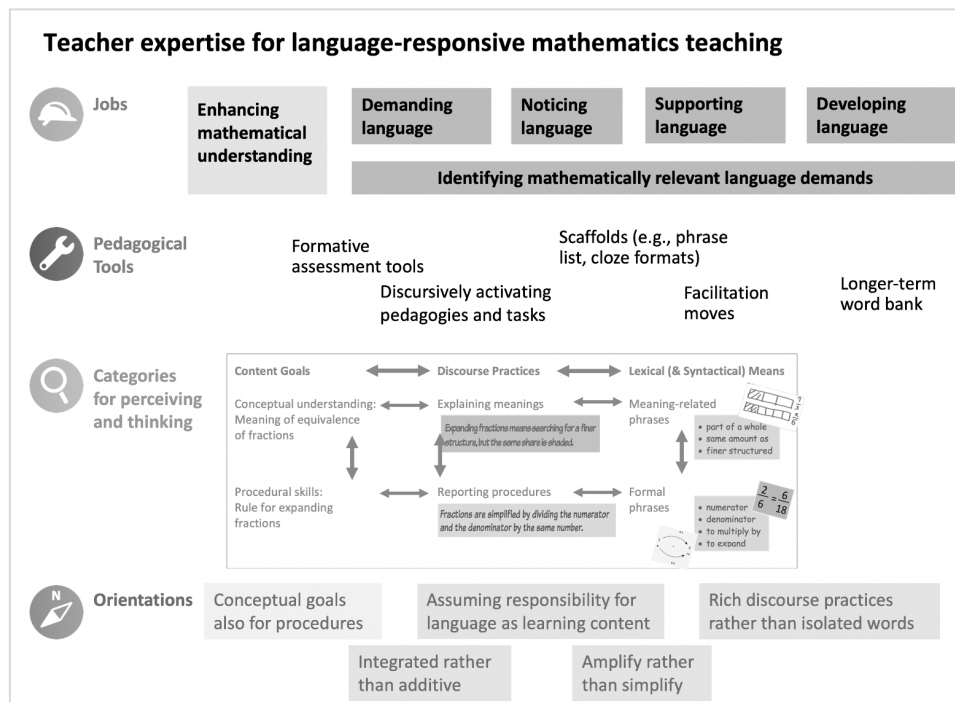


Figure 1. Specifying teacher expertise for language-responsive mathematics teaching (slightly adapted from Prediger, 2019)

Six recurrent jobs of language-responsive mathematics teaching were identified in the existing literature (Grossman et al., 1999; Hajer & Norén, 2017; Lucas & Villegas, 2013) and in design research studies of teachers’ learning processes (Prediger, 2019):

- **ENHANCING UNDERSTANDING** is the overall job of promoting students’ mathematics learning, with a focus on promoting conceptual understanding. All other language-related jobs contribute to this job.
- **DEMANDING LANGUAGE** in cognitively and discursively rich learning situations for requiring language reception (e.g., reading) and eliciting language production (e.g., with pedagogical tools such as writing tasks or activities that initiate student talk).
- **NOTICING LANGUAGE**, that is, noticing students’ language, both in terms of resources and learning needs, in their utterances and written products (e.g., with pedagogical tools for formative assessment).
- **SUPPORTING LANGUAGE** so that students can fulfil demands above their current proficiency level (e.g., with pedagogical tools of written scaffolds or micro-scaffolding moves).
- **DEVELOPING LANGUAGE**, by extending students’ language for the longer-term growth of academic language (e.g., by connecting everyday language to school academic language and technical language and constantly working towards further development)
- **IDENTIFYING LANGUAGE DEMANDS**, that is, identifying mathematically relevant language demands so that the jobs of noticing and supporting can focus on crucial rather than peripheral language demands.

DEMANDING and **SUPPORTING LANGUAGE** are important jobs for all language learning (e.g., Smit et al., 2016). For these jobs to be performed adaptively for each student, **NOTICING**

LANGUAGE is relevant, not only for language-responsive teaching but also generally to all mathematics teaching (Empson & Jacobs, 2008). The longer-term perspective on successively DEVELOPING LANGUAGE builds upon the short-term jobs of DEMANDING, NOTICING, and SUPPORTING, but additionally requires longer-term planning beyond the single lesson to provide language learning trajectories over a teaching unit or a school year (Gibbons, 2002; Lucas & Villegas, 2013).

The fifth job of IDENTIFYING mathematically relevant language demands has been specified as critical for achieving an integration of language and content learning (Hajer & Norén, 2007; Khisty & Chval, 2002; Prediger, 2019), as it underlies the four other language-related jobs. Research has often found that teachers NOTICE, DEMAND and SUPPORT language features that are not central to mathematics learning (e.g., Khisty & Chval, 2002; Moschkovich, 2013), for instance correct language forms (such as orthography) rather than language use. Hence, it is important that teachers focus on the mathematical content learning goals, identify the genres needed to make each of these content goals explicit (e.g., reporting procedures for acquiring procedural skills, and explaining meanings for developing conceptual understanding), and identify the vocabulary and grammatical structures needed to engage in rich discussions (see Prediger, 2019 for details).

The ways teachers prioritise the jobs and master them with specific teaching practices is always implicitly or explicitly guided by their *orientations* (Schoenfeld, 2010). For the jobs of language-responsive mathematics teaching, the following essential orientations were identified (Prediger, 2019; Prediger et al., 2019):

- *Conceptual focus, also for procedures.* Teachers who adopt a conceptual focus usually prioritise the job of enhancing students' understanding, through conceptually focused practices (Beswick, 2007). In contrast, a mainly procedural orientation might lead teachers to re-interpret the job with procedural practices, for example choosing only procedural content goals or offering only procedural exercise tasks. Empirical studies have shown many teachers neglect the conceptual focus, particularly for socially underprivileged students (Boaler, 2002; Wilhelm et al., 2017) or language learners (Adler, 2001; Setati, 2005; Zahner et al., 2012). This neglect of the conceptual focus can be reinforced by textbook authors' suggestions for language learners (de Araujo & Smith, 2022).
- *Assuming responsibility for language as a learning content.* Mathematics teachers may not see themselves as language teachers even though an assumption of responsibility for language as learning content is necessary for teachers to engage in the language-related jobs (Lucas & Villegas, 2013).
- *Amplify rather than simplify.* A focus on language in subject matter classrooms is sometimes misinterpreted as call for simplifying language demands (Lyon 2013). In contrast, an amplifying orientation (Bunch, 2013) seeks to provide students with rich opportunities for engaging in discussions and connecting representations.
- *Focus on rich communication rather than isolated words.* While many teachers have been shown to focus extensively on isolated words (Turner et al., 2019; Zahner et al., 2012), the focus should be on engaging students in rich discussions (Barwell, 2020), with genres such as explaining or arguing (Dominguez, 2011; Moschkovich, 2013)

- *Integrative rather than additive approaches.* Whereas some teachers treat language demands as an addition to and separate from the content learning goals (Lyon, 2013), an orientation in which the language demands are considered as an integrative component of content learning is more productive for achieving mathematical content goals (Prediger, 2019).

3 Extending the jobs and orientations for language-responsive mathematics teaching to multilanguage-responsive mathematics teaching

Having sketched an existing conceptualisation of expertise for language-responsive mathematics teaching, this section now extends this conceptualisation into multilanguage-responsive mathematics teaching. We define *multilanguage-responsive mathematics teaching* as the teaching practices that have the potential to deepen multilingual students' mathematics learning by (a) enhancing students' academic language proficiency (e.g. by building on their informal practices of discussing or arguing and leveraging them) and (b) creating opportunities for students' multiple languages to become a resource for conceptual understanding. As for language-responsive teaching in Section 2, we think of (a) and (b) as *integrated, not additive* practices. In the following, we specify productive practices of multilanguage-responsive mathematics teaching from theoretical understandings about language as a resource (Planas, 2018). For this, we first briefly review important aspects of the rich research on teaching and learning mathematics in multilingual classrooms (Subsection 3.1). This research informs our extension of the jobs (Subsection 3.2) and of the orientations (Subsection 3.3) of multilanguage-responsive mathematics teaching.

3.1 Epistemic potential of multiple languages in classrooms with non-shared multilingualism – state of research on students' learning

The use of multiple languages in classrooms has often been studied in language contexts of shared multilingualism, i.e., in classrooms where some students or all students and the teacher share two or more languages (e.g., Adler, 2001, for South Africa; Barwell, 2020, with French-English in Canada; Moschkovich, 2002, with Spanish-English in the U.S.). These studies revealed not only many communicative benefits of multilingualism, for instance for facilitating students' participation and exchange of ideas, but also the epistemic potential for facilitating students' mathematical meaning-making processes (Barwell 2018; Planas, 2018; Planas 2021).

In other contexts, such as New Zealand, Australia and many European countries, classrooms are shaped by super-diversity with many non-shared languages being present (Clarkson, 2006; Vertovec, 2007). Here, it is not always possible to transfer insights from contexts of shared multilingualism to these super-diverse classrooms. For instance, providing teaching materials in all languages present in a classroom may not be feasible. It has also been noted that some teachers worry about having a lack of control when students work in a language they do not understand (Duarte, 2019). Yet, responding to students' linguistic diversity is crucial to “leverage [their] multiple languages and cultures in instruction”, particularly their languages as meaning-making resources (Daniel & Pray, 2017, p. 789).

However, also in super-diverse classrooms, there are possibilities for making use of the *epistemic potential* of multiple languages as tools for thinking and knowledge construction (Prediger & Uribe, 2021). In these classrooms with only one shared language for communication, multilingual students have been shown to think mathematically in their multiple languages (Clarkson, 2006; Garcia, Johnston & Seltzer, 2017; House & Rehbein, 2004; Lüdi, 2006; Parvanehnezhad & Clarkson, 2008). Therefore, it seems reasonable to anticipate that students' multiple languages can be harnessed for their epistemic potential also in super-diverse classrooms. Strategies for exploiting the epistemic potential of multiple languages include making multiple languages visible (Barwell, 2020), supporting students' active use of multiple languages as (re-)resources for meaning making (Parvanehnezhad & Clarkson, 2008; Setati et al., 2008) or strategies that develop students' mathematical thinking, their languages and the language of instruction in an integrated way (Prediger et al., 2019).

Case studies on students' learning in super-diverse classrooms highlight how facilitating students' use of multiple languages can result in several opportunities for utilizing language as a resource. Firstly, a classroom that is open to students' multiple languages can support students to mobilize appropriate skills from their the linguistic repertoire to respond to a particular task (Sierens & van Avermaet, 2014). This is because they can utilise their higher language awareness (Barwell, 2018) to identify in-the-moment ways to express their thinking (Clarkson, 2006). Secondly, students can use their language as communicative resource to identify and find alternatives to more complex mathematical language, for example by inventing interim vocabulary (Planas, 2021). Similarly, by using multimodal means other than speech and writing, such as graphical representations, students can sidestep grammatical and lexical challenges (Fernandes, et al., 2017). Thirdly, using multiple languages can empower students and support their agency in engaging with mathematics, if activities allow them to use their home cultures and languages as mediational means for sense-making (Noren, 2015, 2008).

Of particular importance for our search for productive practices is the epistemic potential of facilitating students' use of multiple languages, in particular for purposes of conceptual development. One way to harness students' multiple languages for this *epistemic potential* even in classrooms with *non-shared multilingualism* is to engage students in reflecting on the conceptual differences when a concept is explained in different languages (Prediger & Uribe, 2021). This strategy utilises the students' metalinguistic awareness to contribute to their mathematical thinking (Clarkson, 2006).

Figure 2 provides an example of the different ways that languages can express meanings about equivalence, an important algebraic concept. The task is about finding the number of matchsticks in a box when two fictitious students, Ahmet and Anouk, have the same number of matchsticks. Equivalence can be maintained by removing matchsticks from both sides, removing boxes on both sides, or grouping matchsticks and removing groups of matchsticks/boxes simultaneously. In the matchstick task, students can develop an understanding of transformation equivalence in the sense that any manipulation of boxes and matchsticks has to ensure that both students continue to have the same number of matchsticks. Through this, students can reinvent certain equivalence operations in an informal way, such as subtracting (of matchsticks and boxes) or of division (Radford, 2022).

Do you know that teacher and students around the world solve the Matchstick problems in different ways? Below are some suggestions on how to solve the situations in Task 1.1 expressed in different languages, with an example from **Situation D**.

Situation D

Ahmet:



Anouk:



"Kotak dan batang korek api sebaiknya pindah ruas". (Budi, Indonesia)

Translation: The box and the matchsticks should be switching sides.

"De doosjes moeten naar het linkerkant. De lucifers moeten naar het rechterkant. Beide kanten herleiden". (Annerooos, Nederland)

Translation: The boxes must go to the left side. The matchsticks must go to the right side. Reduce both sides.

"Kutular denklemin bir tarafında yalnız bırakılır." (Elif, Turkey)

Translation: The boxes are left alone on one side of equation.

"आपको समीकरण में जैसे ऑपरेशन का इस्तेमाल करके बक्से को अलग करना होगा."

"Aapako sameekaran mein jaise opareshan ka istemaal karake bakse ko alag karana hoga." (Jay, India)

Translation: You must isolate the boxes using the same operation in the two sides of the equation.

- a) Compare the strategies suggested by Budi, Annerooos, Elif and Jay. Which strategy do you think is helpful for you to solve **Situation D**? Which strategy do you think is confusing? Why?

Figure 2: Example task for facilitating conceptual insight through comparing conceptualisations across descriptions in multiple languages

All languages can express transformation equivalence in a range of phrases, while some expressions are more commonly used in a particular language. Within this range, not all phrases are precise in their articulation of equivalence, and different connotations might emerge from them. Accordingly, by asking students to interpret the different speech bubbles to determine how many matchsticks are in a matchbox, students are invited to compare how equivalence is maintained during the suggested transformations. For instance, the Dutch utterance by Anne-roos (from the Netherlands) is common for an operational understanding of equivalence where x has to be in the left side and the "result" has to be on the right side. In the matchstick situation, this operational notion is not viable, because the remaining box will be on the right side with fictitious student Anouk. Accordingly, discussing and comparing the phrases uttered by the fictitious students could engage students in investigating operational and relational understandings of the matchstick situation and so to deepen their conceptual understanding.

3.2 Extended jobs for multilanguage-responsive mathematics teaching

Based on the research on productive practices of using multiple language for mathematics learning, we adapted the five language-related jobs, introduced in Section 2, for the situation of *multilanguage-responsive mathematics teaching*.

- ENHANCING UNDERSTANDING, the central job for language-responsive mathematics teaching is also central for multilanguage-responsive mathematics teaching, as it is a universal goal of promoting students' mathematics learning.
- ELICIT LANGUAGES: The job of DEMANDING LANGUAGE from language-responsive mathematics teaching can be extended to ELICIT LANGUAGES in multilanguage-responsive mathematics teaching. While DEMANDING LANGUAGE also refers to students' language reception and production in the LoI, the multilanguage-responsive teaching job also comprises eliciting students' multiple languages in language production, as part of the requirement to make multiple languages available in the classroom (Barwell, 2020). Teachers need ways to engage students in speaking and writing in multiple languages so that students mobilise their whole language repertoire such that language can become a resource. When there is no one else in the class who speaks the same language, eliciting languages can be realised through specific prompts, e.g., by a homework involving the parents or working with internet sources (Prediger & Uribe, 2021).
- NOTICING LANGUAGES is necessary in all language-responsive mathematics classrooms so that teachers adapt to students' oral or written utterances and support them when necessary (Parsons et al., 2018). For multilanguage-responsive teaching, noticing can include "the attention to how learners use language as they engage with activities like making conjectures, examining constraints, making inferences, abstracting, inventing, explaining, justifying, and challenging" (Essien et al., 2016, p. 107). In classrooms with non-shared languages, noticing practices can be limited when teachers do not understand a student's oral or written language productions. Reflection opportunities resulting from language differences might not be noticed, also because students translate interesting differences only vaguely, without awareness of the potential of discussing subtle differences in connotations (Prediger & Uribe, 2021). Nevertheless, there are still possibilities for teachers to notice how students recognise subtle differences in how different languages conceptualise mathematics, as we illustrate below.
- SUPPORTING LANGUAGES AND LANGUAGE REFLECTION refers to teachers' written or oral discursive and lexical scaffolds provided to support students' engagement in classroom tasks and discussions. In classrooms with non-shared languages, discursive scaffolds (e.g., constantly pressing for "Why?") can be provided in the LoI and still support the elicitation of multiple languages. Written scaffolds in multiple other languages can be provided, e.g., through internet sources, such as online dictionaries, mathematics websites, and YouTube videos. For supporting language comparisons, scaffolds might target language reflections, for example by posing reflection questions in tasks that specifically ask students to consider how different languages emphasise different aspects of a concept. For example, in different languages, the word for triangle highlights different aspects of the shape ('tri-angle' in

French refers to the angles, ‘Drei-eck’ in German refers to the vertices, ‘tre-kant’ in Norwegian refers to the edges). Students could be asked to reflect how the names constitute different understandings about triangles.

- **SUCCESSIVELY ENHANCING LANGUAGES** is the job in which (a) a sequence of language learning opportunities is planned, (b) the teachers make use of students’ multilingual and multimodal utterances in productive ways, and (c) valuable ideas and contributions are re-voiced and connected to more concise language. All of these elements are also crucial for multilanguage-responsive classrooms (Essien et al., 2016), with an additional focus on language awareness for subtle differences in connotations: As teachers’ possibilities of leveraging students’ languages can be limited when there are many non-shared languages, the enhancement focus can also be laid on (c) growing students’ language awareness, so that students can increasingly engage in deep reflections.
- **IDENTIFYING MATHEMATICALLY RELEVANT LANGUAGE ASPECTS** focuses on bringing different mathematical meanings into view. In multilanguage-responsive teaching, this job involves identifying opportunities for comparing ways of expressing the same mathematical idea in different languages or the same language, that is, identifying possible epistemic potential for conceptual understanding. Whereas sometimes the opportunities occur spontaneously in classroom discussions (Barwell, 2020; Planas, 2018), teachers can also prepare situations which deliberately guide students to value how such differences can enrich mathematical understandings, for example by asking targeted questions when a task provides an opportunity for comparing different mathematical meanings.

For exemplifying productive teaching practices for responding to these jobs, we imagine how teachers could implement the task on equivalence in Figure 2 in a multilingual classroom. For example, they might **ELICIT LANGUAGES** by inviting students to watch YouTube videos in other languages to investigate how transformation equivalence is conceptualised in these languages. The speech bubbles in the task can support teachers to facilitate the students’ reflection on how equivalence is expressed in different ways. Such reflections can lead students to recognise that some phrases are better suited for expressing a relational understanding of the matchstick operations. However, an expert teacher will **NOTICE** which student only treats the task superficially, and those who become **AWARE** of the subtle differences in the language-specific conceptualisations of equivalence, because only then can the reflection contribute to students **DEVELOPING** relational **UNDERSTANDING** of equivalence. This requires that the teachers had **IDENTIFIED CONCEPTUALLY RELEVANT LANGUAGE ASPECTS**, e.g. that “Pindah ruas / switching sides” or “naar de linkerkant brengen / bring to the left” are expressing inappropriate operational meanings of equivalence. Instead, the key idea is not yet expressed in any of the bubbles: “when treating an equation, it should still keep the same balance, by only changing the same on both sides”. Perhaps, students will not yet be able to talk about “the same balance”, so this is the meaning-related language that is to be developed by **SUCCESSIVELY ENHANCING LANGUAGES**. To achieve this, the teacher needs to leverage students’ ideas about mathematically appropriate way of describing equivalence transformation, into a concise but not yet formal language. For instance, students could extend the utterance in Hindi (Jay, India) to describe the matchstick operations in a relational way, for example in terms of “removing the same number of matchsticks *on both sides* ensures that both students, Ahmet and Anouk, still have

the same number of matchsticks.” A teacher who misses this critical conceptual aspect will not be able to exploit the epistemic potential in a deep manner. This highlights the need for appropriate professional development, which will be discussed in the following subsections.

3.3 Productive orientations for multilanguage-responsive mathematics teaching

Since the early work on multilingual classrooms, researchers have emphasised that teachers’ practices of including and using multiple languages depend on their orientations towards language policies, which can lead teachers to favour politically valuable languages instead of home languages (e.g., Planas & Civil, 2013; Setati & Adler, 2001). Therefore, teacher educators aim at developing teachers’ “understanding of the social, political and economic struggles surrounding the use of the two languages.” (Garcia, 2008, p. 388). While orientations around language policies, which may hinder students’ learning of mathematics have been discussed intensively (e.g., Essien et al., 2016; García, 2008), only a few studies have contributed to explicitly investigate teachers’ productive orientations that underly their practices for exploiting multiple languages as resources with respect to communicative and epistemic potential (Essien et al., 2016). Therefore, to identify valuable orientations, we draw upon previous research on multilingual classroom practices for all six jobs (Barwell et al., 2016).

3.3.1 Resource orientation rather than deficit orientation: Languages as resources

A *resource orientation rather than deficit orientation* has often been discussed since the 1980s (Ruíz, 1984; Secada, 1992) as a precondition for teachers to take on any of the five multilanguage-responsive teaching jobs in that they value their students’ multiple languages as resource, for communication and learning (Essien et al., 2016; Planas & Civil, 2013; Setati & Adler, 2001). The resource orientation of multiple languages was originally articulated for language planning policies (Ruíz, 1984) and has been increasingly transferred to classrooms (Planas, 2018; Secada, 1992). In educational contexts, the resource orientation has been challenged by frequent concerns that the use of multiple languages might reduce the time for learning the LoI or mathematics (Sierens & van Avermeat, 2014). However, the potential for mathematics learning to be improved through the use of multiple languages has been shown in several studies (Barwell et al., 2016; with quantitative evidence Schüler-Meyer et al., 2019).

Teachers who regard languages as learning resources can become aware of opportunities for utilizing students’ languages in mathematical learning processes and ELICIT AND SUPPORT LANGUAGES for this purpose (Barwell, 2020; Planas, 2018). In particular, ELICITING is tied to this orientation, because the proactive elicitation of multiple languages only makes sense when languages are seen not as disruptive, but as a resource to build upon. So, the resource orientation can best unfold into *productive teaching practices* when being combined with further orientations which are more specific to mathematics education.

3.3.2 Conceptual orientation rather than procedural orientation: Connecting the goals

A *conceptual rather than procedural orientation* entails that the overall goal of mathematics teaching is to promote students’ conceptual understanding of mathematical concepts and to connect procedures to that conceptual understanding, so to take the job ENHANCE UNDERSTANDING seriously (with the research backgrounds explored already in Section 2).

3.3.3 Integrative rather than additive orientation: Epistemic potential of multiple languages

The *integrative orientation* entails that language learning is not considered separate and in addition to content learning, but as highly intertwined. Accordingly, teachers holding an integrative orientation usually integrate two jobs, ENHANCING UNDERSTANDING and SUCCESSIVELY DEVELOPING LANGUAGE (Lyon, 2013; Prediger, 2019).

With respect to multiple languages, a second integration becomes important, namely combining the first two orientations, pushing conceptual learning goals with languages as resources (Essien et al., 2016). So far, many teachers and researchers have mainly focused on the communicative potential of multiple languages, in particular for increased participation in communication or identity development (Sierens & van Avermeat, 2014). Beyond this, an integrative orientation toward language in mathematics learning can contribute to teachers exploring the *epistemic potential of multiple languages* (see Section 3.1). Similarly, understanding the epistemic potential of multiple languages will enable teachers to integrate language learning goals with conceptual learning goals.

The integrative orientation is the starting point for developing teachers' expertise for harnessing the epistemic potential of multiple languages in a PD program and to realise the job IDENTIFYING THE MATHEMATICALLY RELEVANT LANGUAGE ASPECTS. The conceptual benefit of multiple languages can take many forms. Firstly, if some students share multiple languages and speak them fluently, they can switch languages in small group discussions to make progress during mathematical problem solving (Clarkson, 2006). In these small group settings of shared multilingualism, students can use their full multilingual repertoire such as gestures, representations, and speech, also in their home language to develop mathematical ideas and explanations (Moschkovich, 2002), and if the teachers share their languages, they can unify these individual notions and inscriptions towards viable notions and representations (Planas, 2021). In teaching-learning units specifically designed for enacting multiple languages in (small group) settings of shared multilingualism, such as systematically connecting representations and registers (Moschkovich, 2013), students' use of their home language can facilitate conceptual insight (Schüler-Meyer et al., 2019).

Furthermore, also in settings of non-shared multilingualism, reflecting multiple phrases for the same ideas (in one or multiple languages) can also allow students to contrast different conceptual connotations, which can result in a more integrated, multi-faceted conceptual understanding (Prediger et al., 2019; Prediger & Uribe, 2021). In these cases, teaching-learning units can facilitate the use of students' multiple languages for conceptual understandings if teachers SUPPORT STUDENTS' LANGUAGE REFLECTION.

Summing up, exploiting the epistemic potential of students' multiple languages is strongly connected to a combination of three orientations. Firstly, as the exploitation cannot be realised by teachers who hold deficit orientations but by teachers who recognise potentials, it is the *resource orientation* which can accelerate teachers' recognition of potential for conceptual insight in other languages than the LoI. While procedurally focused classrooms need much less language and language reflection, it is, secondly, teachers' *conceptual orientation* by which they put emphasis on those learning goals for which language is a catalyst. Thirdly, the epistemic potential can hardly be harnessed by teachers who hold additive orientations because the idea of epistemic potential in itself is tied to an integrative orientation. But even if teachers

hold resource-focused, conceptual, and integrative orientations, their practices for implementing the jobs of ENHANCING UNDERSTANDING and SUCCESSIVELY DEVELOPING LANGUAGES are influenced by two further orientations.

3.3.4 Amplifying orientation rather than simplifying orientation: Pushing beyond resources

In many multilingual classrooms, teachers have been found to first intuitively start their language-related approaches by simplifying their use of language, by reducing language demands, or by overly directive scaffolds (Lyon, 2013; Turner et al., 2019). Within such a simplifying orientation, the use of home languages can be considered mainly as a simplifier (Duarte, 2019). In contrast, an *amplifying orientation* in multilanguage-responsive teaching draws upon students' multiple languages as means to enrich and deepen students' languaging and learning, and with the goal to amplify also students' LoI use in the long run. Given that in some countries, some languages are considered more useful, some students do not use their home languages in self-initiated ways in the beginning, so amplifying starts with ELICITING STUDENTS' LANGUAGE and SUPPORTING not only the language use, but also deepening the LANGUAGE REFLECTION. Beyond using language as resource, amplifying orientations can also lead teachers to SUCCESSIVELY DEVELOP STUDENTS' MULTIPLE LANGUAGES.

As a consequence, PD programs can create opportunities for teachers to respond to these jobs in an amplifying orientation, by learning to create student opportunities for language production and translanguaging, with strategies such as making languages visible, for instance through attending to and analysing of specific ways of talking (Barwell, 2020), and to facilitate "the deliberate, proactive and strategic use" of students' home languages (Setati et al., 2008).

3.3.5 Discursive rather than lexical orientation: Dynamic languaging

Engaging in the job of SUPPORTING LANGUAGES, particularly with the goal of utilising the epistemic potential of multiple languages, requires teachers to understand how the use of multiple languages is discursive and dynamic. As Moschkovich (2002) noted two decades ago, successful teachers have moved beyond just making students' multiple languages visible, for example, through providing translations for words or definitions. Instead, the inclusion of multiple languages requires facilitating dynamic translanguaging practices with their focus on the discursive level. For instance, mathematical problems that connect to everyday problems that students discuss in their home languages allow these students to make use of their multilingual repertoires by extending their everyday practices of discussing, arguing, or exploring into more mathematical practices (Domínguez, 2011). Similarly, in rich discursive practices, students can use further communicative resources such as gestures and representations, to support their communication in LoI (Moschkovich, 2002). Overall, research in multilingual mathematics learning repeatedly suggests to create strong relations between rich mathematical practices and students' use of their multilingual repertoires as resources in PD programs.

In particular, for the job of IDENTIFYING MATHEMATICALLY RELEVANT LANGUAGE ASPECTS, the discursive rather than lexical orientation has been shown to be a crucial content in PD courses (Moschkovich, 2013; Prediger, 2019): Teachers who focus on how students explain meanings, argue about their ideas, and report procedures, have been shown to NOTICE more relevant emergent language aspects that are worth to be SUPPORTED in the moment (Prediger

et al., 2019), and hopefully for SUCCESSIVELY DEVELOPING them in the future. Further research will be needed to investigate in how far these connections also apply to other aspects of dynamic languaging.

4. Discussion

In this paper, we have highlighted the need for professional development in order for teachers to harness the epistemic potential of multilingualism to deepen conceptual understandings in mathematics. For this purpose, we have introduced the notion of *multilanguage-responsive mathematics teaching* and worked through the existing state of research on mathematics teaching and learning in multilingual classrooms to determine how the jobs of language-responsive teaching can be modified with respect to the epistemic potential of using and reflecting multiple languages. To understand what might push or hinder teachers to respond to these jobs, we have condensed existing research to describe five productive orientations that can implicitly or explicitly guide teachers' perceptions and job priorities in fruitful ways.

In Figure 3, we summarise the specified, interrelated jobs and orientations of multilanguage-responsive teaching. While many of them are related to language-responsive teaching in the LoI, the most specific job of multilanguage-responsive teaching is to IDENTIFY MATHEMATICALLY RELEVANT LANGUAGE ASPECTS, because this contributes to teachers harnessing the epistemic potential of multilingualism. Particularly, contrasting multiple languages can highlight specific nuances of a mathematical concepts and hence, when harnessed as learning opportunity, can facilitate students' conceptual insight.

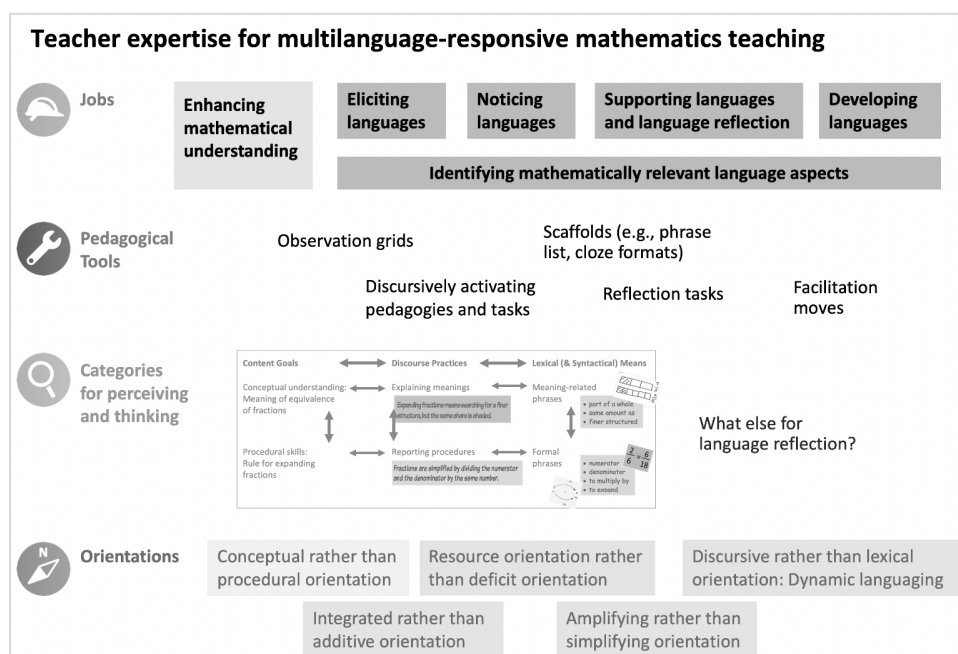


Figure 3. Specified teacher expertise for multilanguage-responsive mathematics teaching (substantially adapted from Prediger, 2019)

While this paper lays a foundation for designing a PD environment by specifying the PD content in more detail, the not-yet-updated categories and tools in Fig. 3 illustrate that more research is needed to better understand what exactly well-targeted PD programs should offer to teachers. Simply specifying orientations and categories will not provide information about how to engage teachers in adopting them in PD courses, since orientations are not formed or changed by simple instruction. Future PD design research should also investigate how teachers can be supported to successively develop their practices to realise the jobs of multilanguage-responsive mathematics teaching.

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